# Inspector guidance: primary mathematics

## The purpose of this document

This document has been created for training and supporting inspectors to conduct subject deep dives in schools. The training guidance provides a structure to explain variation in subject-level impact. It should be used in conjunction with handbooks for section 5, section 8 inspections of good and outstanding schools, and section 8 no formal designation (subject-specific) inspections.

**Points to consider when examining the evidence:**

School leaders may not be able and should not be expected to articulate their intent **as it is outlined** in this document or to provide documents which neatly provide the evidence for these focus areas.

Inspectors should always investigate claims that issues affecting quality of subject education are outside the school's control. It should be evident that the issue has been identified prior to the inspection and that the school has taken steps to mitigate the ill effects. For example, in the case of text books or materials downloaded from websites, it should be clear that leaders have previously identified the issue and raised it with senior leadership, investigated funding, identified texts they would prefer, identified the specific weaknesses of the current text and taken specific action to mitigate against those weaknesses.

## The six focus areas

The six focus areas below provide a structure to explain reasons for the quality of subject education as identified by inspection activities. Inspection activities are likely to be an iterative process as inspectors consider evidence of impact and evidence which explains that impact. Under each focus area are three columns.

**Column 1:** This provides examples of useful questions inspectors might ask of people or evidence to explain reasons for the quality of subject education. This is **not** a comprehensive list of questions which may be asked. Inspectors should use their own judgement but will find these suggestions useful.

**Column 2:** This is an outline of potentially stronger practice in the area each question explores.

**Column 3:** This is an outline of weaker practice in the area each question explores. It also provides likely responses and other evidence inspectors may hear or encounter and gives explicit guidance on how these responses can be interpreted.

**Inspectors are likely to use the following sources of evidence in making their judgements:**

**<u>They will generally use:</u>**

- interviews with subject lead (if there is one) and/or the appropriate senior leader such as the headteacher
- curriculum plans
- pupils' work
- discussions with pupils
- interviews with teachers
- lesson visits, including conversation with teachers, if possible.

**<u>Where appropriate, inspectors may use:</u>**

- the school's own records of lesson visits in the subject
- the resources available for teaching mathematics (incl. school library, ICT facilities (access to GIS))
- the school's assessment policy
- assessment instruments, including mark schemes if there are any (not internal data)
- how the school provides pupils with feedback on their work
- how the school promotes the value of the subject, including via enrichment activities
- forms of support for inexperienced, non-specialist or struggling staff
- any support provided for the subject lead
- performance management's role in improving mathematics provision
- details of the timetable and staffing (including details of experience and qualifications of staff)
- school policies on teaching, assessment, homework and behaviour
- documents analysing strengths and weaknesses of the subject and any associated improvement plans.

# Contents

## 1. The school's understanding of progress in mathematics and how that informs its approach to the curriculum

| Focused questions to ask the evidence | Outline of potentially stronger practice in terms of intent, implementation and impact<br>NB: answers will take many forms. Below are common findings to look out for. | Outline of weaker practice in terms of intent, implementation and impact<br>NB: answers will take many forms. Below are common findings to look out for. |
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| 1) Scope: Does subject planning give meaningful attention to all categories of knowledge in which progress is made (is this commensurate with that outlined in the NC)?<br><br>**School friendly questions**:<br>How do you ensure that planning identifies all the mathematical facts, methods and strategies that pupils need to learn? | **A curriculum that engineers success**<br>Given the hierarchical nature of mathematics, a systematic approach to planning is ideal. A great curriculum engineers success by incorporating the granular detail, sequencing and 'bigger picture' of curriculum content.<br>The curriculum plan should identify the powerful core knowledge that pupils need to take with them on their journey into the next year, phase and stage of their mathematics education.<br>The categories of content considered here are declarative, procedural and conditional knowledge. These categories are then further broken down into sub categories as illustrated in the table below: | **Loose topic planning and home-made resources predominate curricular strategy**<br>Non-specialists or inexperienced teachers will need clear guidance in what they teach and the best resources to use. Reputable schemes may help. The school must ensure that all resources are used in a logical sequence.<br>**The school policy is for teachers to re-design curriculum plans based on the individual needs of their classes and not rely commercial schemes of learning or associated textbooks**<br>There is no automatic merit in continuously re-designing curriculum plans for the sake of it. Such an approach not only drives increasing and unnecessary workload, but also neglects the benefits of iterative approaches that seek to improve plans over time based on the similarities between pupils and their likely acquisition of core content. |

| Content categories | Type I | Type II |
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| **Declarative** | Facts, formulae | Knowledge of *relationships* between facts/concepts |
| **Procedural** | Methods, algorithms | Knowledge of *relationships* between facts, steps and missing facts/principles & mechanisms |
| **Conditional** | Strategies (PK + DK) | Knowledge of *relationships* between known information, strategy choices and unknown information/reasoning |

2) Scope and components: Does planning ensure that pupils learn the most important facts and concepts?

**School friendly question:**

How have you identified the most important facts and formulae for pupils to learn?

**Declarative knowledge** includes the facts of number, time and space: number bonds, times tables, formulae and rules, for example. This category also includes knowledge of *relationships* between facts. These are the concepts that link facts together in 'families' or familiar patterns, that give pupils the ability to *understand* as well as the ability to recall mathematical information with precision and clarity. The litmus test for this category of knowledge is the use of the sentence stem '**I know that**'.

**Pupils need to know, to automaticity, useful facts & formulae:**

**KS1 examples**

***'I know that two eights make sixteen'***

***'I know that there are a hundred pennies in a pound'***

***'I know that a shape with five sides is called a pentagon'***

**KS2 examples**

**The school's policy is to provide learning aids to circumvent lack of automaticity**

Enabling pupils to bypass learning core content, such as through using a multiplication grid because pupils have not learned useful maths facts, risks pupils being unable to access content later.

**Pupils can't readily use prior learning in more complex tasks: they forget steps, work slowly, are only be able to use easy numbers, or make repeated errors**

One, some or all of the identified elements signal lack of automaticity in knowledge of components which needed to be identified, taught and rehearsed.

**Pupils who lack familiarity with maths facts are encouraged to choose from a variety of physical resources, such as dienes blocks, to help them understand questions**

Prompts, scaffolds and manipulatives should, ideally, be used to help explain underlying principles rather than make up for lack of knowledge of facts and methods of calculation. If the aids are used to make up for lack of knowledge, then pupils may become reliant on them. Furthermore, making choices, being

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| | ***'I know that nine squared is eighty one'*** <br><br> ***I know the convention for writing numbers larger than a million'*** <br><br> ***I know that angles in a triangle sum to 180 degrees'*** <br><br> A carefully considered and sequenced curriculum should ensure all pupils are given the most useful number facts, conventions, formulae and vocabulary in all topics covered – their acquisition should be ordered so that there are opportunities to both learn and to use this knowledge until both speed and accuracy of recall are achieved. <br><br> **Pupils need to know concepts:** <br><br> **'I know that if….., then……'** <br><br> A carefully considered and sequenced curriculum should also detail the intended learning of patterns, principles and rules, often referred to as 'conceptual knowledge' or 'number sense', that may result from familiarity with certain calculations as well as from teacher exposition. For example, being taught the pattern that adding 9 will result in an increase of 1 in the tens and a decrease of 1 in the ones column. | distracted by colours and textures of manipulatives as well as a greater number of steps to finding a solution introduces a greater possibility of errors which may then become permanent misconceptions. |
| 3) Scope and components: Does planning ensure that pupils learn the most efficient and accurate methods? | **Procedural knowledge** is fluid in nature and includes systematic methods, algorithms or procedures; everything from long division, ways of setting out calculations in workbooks to the familiar step-by-step approaches to solving quadratic equations. Like declarative knowledge, | **The school's policy is for pupils to develop their own methods of calculation, in order to further develop and show understanding** <br><br> This expectation may work for previously high attaining pupils who have sufficient accurate knowledge to design their own processes, but many |

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| **School friendly question:**<br>How have you identified the most useful methods for calculation that pupils will learn? | the category also branches into type II knowledge of relationships, but this time what is known are the links between the process or steps and the information presented in a question. This knowledge, once learned, enables a pupil to process mathematical information speedily and accurately. The litmus test for this category of knowledge is the use of the sentence stem '**I know how'**.<br>**Pupils need to know, to automaticity, useful, efficient and accurate methods of calculation:**<br>**'I know how…'**<br>A carefully considered and sequenced curriculum should ensure all pupils are given the most efficient and useful procedures for all aspects of finding solutions in sets of exercises and problems – these procedures should enable swift and accurate calculation and would be considered important components for success in the next phase of mathematics education.<br>The curriculum should aim for both understanding of underlying principles as well as speed and accuracy of use, with each aspect leveraging the other's development.<br>**Presentation of calculations:**<br>Standard methods for laying out bookwork may also constitute part of the school's calculaton and presentation policy, so that pupils are more likely | pupils risk developing strategies that are inefficient and more likely to lead to errors. Pupils are unlikely to, for example, discover short division, yet this is the most efficient method that they would need for success in their next stage of education.<br>**The school's policy is that pupils are allowed to learn the formal methods, such as column addition, once they have demonstrated understanding of the underlying concepts through use of more informal and expansive methods of calculation**<br>Teachers need to consider whether the expectation of written use of informal or mental calculation methods as a way of showing understanding risks pupils being held back and limited to inefficient methods that increase the likelihood of error while obfuscate emerging patterns.<br>**Pupils do not know any formal methods**<br>Pupils who are not equipped with formal methods of calculation, including their layout, might find the transition to year 7 difficult because these methods will be applied within the context of more complex mathematics. |

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| | to avoid errors, see patterns and experience success.<br><br>The curriculum should aim for both understanding of underlying principles as well as speed and accuracy of use, with each aspect supporting the development of the other. | |
| 4) Scope and components: Does planning ensure that pupils learn useful strategies for solving problems?<br><br>**School friendly question:**<br><br>How have you identified the most useful problem-solving strategies that pupils will learn?<br><br>How have you planned for pupils to gain familiarity with problem types so that they can easily identify strategies to use? | **Conditional knowledge** gives pupils the ability to reason and solve those problems where both the answer and the process for finding the answer are not immediately apparent. Strategies are combinations of facts and methods that can be paired to types of problems. Recognition of the deep structures of problems and their relation to strategies is also crucial content that must be learned. The litmus test for conditional knowledge is the use of the sentence stem **'I know when'**.<br><br>**Pupils need to know topic-specific strategies for problem solving and be familiar with their conditions of use**<br><br>**To avoid cognitive overload, pupils need to have previously attained automaticity in the linked facts and methods**<br><br>**'I know when…'**<br><br>A carefully considered and sequenced curriculum should ensure all pupils are given strategies to solve problems. Combinations of facts and methods are transformed into strategies when pupils gain familiarity with the types of | **The school encourages pupils to take a creative approach to problem solving**<br><br>Analysis of expert problem solving, despite the appearance of creativity, shows that they are systematic, drawing on extensive prior knowledge. Creativity in problem solving without the facts and methods required to problem solve risks pupils making mistakes or accumulating misconceptions.<br><br>**The school set problem-solving activities for every lesson**<br><br>New problem-solving or mathematical reasoning activities should only be set *after* pupils have the relevant prior mathematical understanding and automatisation of associated facts, concepts, procedures and strategies. Furthermore, novices need guidance to identify an underlying pattern or deep structure of the problem, instead of just noticing the superficial features.<br><br>**The school believes pupils become better 'problem solvers' through engaging with open-ended problem solving activities**<br><br>While there are some useful generic strategies pupils can learn, there is no structure or strategy that will make pupils significantly better at solving non-routine |

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| | questions, exericises and problems that these strategies can be used for.<br><br>**Problems vs. Exercises:**<br><br>The definition of problems in this context are those questions where both the process and the solution are unknown. Exercises refer to questions where the procedure is pre-defined.<br><br>**Content vs. activity:**<br><br>In mathematics, the 'problems' that pupils are asked to solve are a separate consideration to the (use of) 'problem solving' as a type of activity-based pedagogy. Problems may be:<br><br>▪ **Abstract** or purely mathematical **problems**<br>▪ **Real world problems**<br>▪ **Routine or non-routine** problems, which can be either abstract or real-world<br><br>Even when a lesson consists entirely of practising material from a previous lesson, pupils will still be making use of logical thought processes to connect and make sense of concepts (rather than accumulating misconceptions). In this sense, developing expertise in reasoning or problem solving will be integral to all lessons.<br><br>**Component parts for conditional knowledge:**<br><br>A curriculum should identify the types of problems a teacher wants pupils to learn to solve, ensuring that pupils have: | problems, in the absence of identifying, sequencing and teaching of strategies for solving particular types of problems (conditional knowledge). A pupil skilled in solving problems of one type, will still have a low probability of success solving problems of another, unfamiliar, type.<br><br>**The school identifies gaps in pupils' problem-solving skills**<br><br>Identification of a gap in a particular generic skill cannot differentiate between lack of automaticity in facts and methods, lack of ability to identify the deep structure of a problem, or lack of knowledge of a strategy or approach to solving that problem type. There is no generic mathematical 'problem-solving skill'.<br><br>**The school's policy is for problem solving to be offered as a 'challenge' and taken up via pupil choice, as part of a wider strategy to offer differentiated activities in each lesson**<br><br>Pupils who are already confident and successful mathematicians may thrive on the offer of problem-solving challenges as something to work towards and then receive extra group teaching for during the course of lessons. For these pupils, there is still a risk of rushing through content and not commiting component parts to long-term memory, thus any 'progress' demonstrated may be built on shaky foundations. For pupils who need more instruction, guidance and practice, choosing problem-solving challenges may lead to frustration, error and accumulation of misconceptions. Those pupils who do |

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| | ▪ Knowledge of how to identify and separate out deep structures of problems from the superficial features<br>▪ Understanding of, and automaticity using, useful mathematical facts, concepts and methods<br>▪ Familiarity with and knowledge of strategies for *types* of problems that have a similar deep structure<br>▪ When applicable, metacognitive strategies and habits e.g. self-monitoring and use of visual representations<br>**Rationales for planning problem solving into a specific curricular sequence**:<br>▪ A likely need to solve similar problems in the future<br>▪ To elucidate connections between previously identified mathematical ideas (this is a less efficient strategy than 'purposeful practice' using near transfer)<br>▪ Offering a sense of what mathematicians do<br>▪ To make firm and precise in the mind the strategies for solving particular sub-types of problems | not choose problem solving may also be inadvertently narrowing their experience of the mathematics curriculum, particularly if the curriculum does not specify and order the teaching of strategies for particular types of problems in other parts of the topic sequence.<br>**Pupils in year 6 struggle to solve word problems**<br>Teachers should be aware that trickier word problems presented to year 6 pupils may require them to manipulate algebraic expressions and solve linear equations in the absence of basic knowledge of algebra. Care should be taken to ensure that pupils have the pre-requisite knowledge to solve such problems, rather than be expected to solve them through trial and error or informal approaches. |
| 5) Components/sequencing: Does planning consider component content and its sequencing to build knowledge | **A curriculum that engineers successful learning of new content**<br>**Within the lesson sequence:** | **The school policy is to ensure learning in each lesson is evident**<br>The progression from simple to complex does not necessarily fit neatly into a lesson, and it is appropriate for teachers to spend lessons recapping prior material, |

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| over time and create 'readiness for future learning':<br>a) within the lesson sequence?<br>b) within the topic?<br>c) within the year or phase?<br>Is planning for 'challenge' understood as building more knowledge over time towards ambitious curriculum end points?<br><br>**School friendly question:**<br>How have you ensured that when pupils encounter new content, they have all the linked facts and methods that are necessary for working with that new content?<br>How do you ensure that further challenge for pupils involves facts, methods and strategies that they know already, rather than content yet to be taught? | Topics should be broken down into manageable steps. Each lesson should have a clearly defined and specific focus, ensuring that the ratio of new to previously learned (linked) content is weighted towards the latter, with no automatic expectation that every lesson should include new content.<br>**Within the topic:**<br>The sequence in each topic should follow the same logic as the curriculum progression itself: procedural knowledge and the relevant conceptual knowledge should be developed in tandem, and both should feed into mathematical reasoning and problem-solving tasks identified as appropriate for pupils given their prior knowledge.<br>A sequence of lessons should be planned in a similarly coherent, logical order so that the mathematics can be connected and build a deep and holistic understanding of the concept. Teachers should ensure that all instruction is clear, concise, accurate and free of unnecessary (and therefore distracting) context.<br>**Within the year or phase:**<br>In order to engineer further success, the planning of pupils' moments to use and apply new component parts (including conceptual links) should be closely aligned and related to the sequence of teaching within and across lessons. Guided practice of similar examples may bridge the gap between receiving new | allocate time to over-learning through purposeful practice, or spend longer than one lesson building towards more complex knowledge. Ofsted does not look for overt progress within one lesson. Manipulating and artificially showing 'progress' through moving quickly through content risks dysfluency and the accumulation of misconceptions over time, particularly for at-risk pupils.<br>**The school policy is that every lesson should include reasoning and problem solving**<br>This is not necessary or even desirable. It is fine to have lessons where the entire time is spent practising something that was taught in the previous lesson. Teachers should be able to justify the goals of such practice, e.g. how automaticity in what is practised will empower specific subsequent learning.<br>**The school uses open-ended problem solving to teach mathematics**<br>Minimally guided instruction is most successful when pupils already have considerable expertise and younger pupils, by definition, are not experts. Without a good base of knowledge, pupils risk rehearsal of error and misconceptions<br>**The school uses investigations as a vehicle for revealing and learning new core content**<br>If the intention is that pupils learn, for example, a key formula such as the area of a rectangle, teachers should be aware that spending the majority of the lesson devoting thought and attention to 'finding out' will lead to aspects of 'finding out' being remembered. This gives rise to pupils remembering the surface |

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| | information and using new information independently. | features of a lesson (the activity itself), rather than the core knowledge intended to be learned. In this example, the key formula that is revealed momentarily, at the end of a lesson, will probably not endure in the minds of pupils. |
| 6) Rigour: Are pupils given the capacity to consider subject-specific questions for themselves by combining knowledge from different aspects of the subject curriculum?<br><br>**School friendly questions:**<br><br>How have you planned the opportunities, such as with varying exercises, for pupils to apply linked facts and methods for themselves?<br><br>How do you design opportunities for pupils to develop new connections of knowledge when working with facts and methods they already know? | **A curriculum that engineers successful opportunities to make links within and between content**<br><br>**Connecting concepts:**<br><br>The curriculum should ensure that new content is easily assimilated because connections with prior component knowledge are already in place. These connections should be planned, explained and repeated within and across topics, and as pupils readily apply previously learned *components* in more complex tasks, both new and previously learned components should be made firm and precise in the mind – the influences are bidirectional and increasing familiarity further elucidates helpful patterns and understanding.<br><br>**Ruling out misconceptions:**<br><br>The curriculum should also anticipate, specify and proactively rule out the accumulation of common misconceptions (erroneous connections) that may inhibit future learning, success and motivation. An example of a common misconception is that pupils misinterpret the equals sign as being operational rather than a symbol of mathematical equivalence. | **Each lesson addresses a different aspect of the topic and so over the period everything is covered**<br><br>It is insufficient to just 'cover everything'. The order and detail in which aspects are addressed is critical.<br><br>**The school views 'understanding' as being analogous to learning**<br><br>A curricular approach that overly emphasises the teaching of the *why* of a procedure, concept or rule and which expects pupils' follow up practice to be predominated by demonstrating proof of 'understanding' will not guarantee that pupils learn useful facts, methods and strategies. Moments of understanding, no matter how powerful, are likely to be fleeting. Pupils need to be taught and then practise the core facts, methods and strategies that, through their use, enable the development of understanding because pupils can witness the new connections and relationships for themselves. |

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| 7) Memory: Is crucial content identified, emphasised and repeated so that pupils know more and remember more (make progress)?<br><br>**School friendly question:**<br><br>How do you decide the quantity, type and timing of linked practice that pupils need to commit taught facts, methods and strategies to long-term memory?<br><br>How do your plans prevent pupils from forgetting what was taught and learned last week/term/year? | **A curriculum that engineers successful consolidation of content**<br><br>A carefully considered and sequenced curriculum should systematically provide opportunities for retrieval, review and overlearning. Many repetitions of learned content are required for overlearning.<br><br>**Retrieval and review:**<br><br>Previous taught concepts should be regularly reviewed, their spacing being carefully considered and planned into the curriculum. Review should prompt pupils to retrieve knowledge from long-term memory because this is more effective for memory than merely re-studying material.<br><br>Application of prior knowledge, providing prior knowedge has been learned to automaticity, to more complex tasks may be an effective for consolidation and also for making links across mathematical topics.<br><br>**Overlearning:**<br><br>Overlearning works to prevent gaps in learning opening up for groups of pupils who would be considered at risk. | **Pupils typically only practise a few examples before moving on**<br><br>Overlearning to automaticity will generally require a large volume of practise examples, particularly for novices. If these are not frequently provided this will probably explain a lack of automaticity.<br><br>**"Teachers do this as part of their teaching"**<br><br>Does the scheme of work actually allow time and is there evidence in pupil work of such consolidation?<br><br>**The school uses investigations as a vehicle both for revision and for linking parts of mathematics together**<br><br>Teachers should consider the level of mathematical support required on planned 'investigative projects' and whether this support may negate any effect of using such projects as consolidation.<br><br>**Pupils' opportunity to recall material is limited to termly tests**<br><br>Expecting pupils to remember briefly encountered material in a yearly test is not enough to help pupils to commit material to long-term memory. |
| 8) Memory: how are pupils' gaps in knowledge of crucial content identified, emphasised and repeated? | **Preventing gaps in learning opening up in the first place, as well as seeking out and closing gaps in learning**<br><br>**Closing the school-entry knowledge gap:** | **"We address these deficits in lessons"**<br><br>Does the curriculum structure allow for this or in practice is addressing gaps subordinate to coverage? Is it evident from lessons or from planning? |

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| **School friendly questions:**<br>How do your plans prevent knowledge gaps of key facts and methods from opening up?<br>How do your plans provide for the identification and repairing knowledge gaps for key facts and methods? | Schools should work towards closing school-entry knowledge and retention gaps, to give at-risk pupils a greater chance of success and avoid expensive and inefficient use of resources for interventions in later year groups.<br>**Identifying missing component parts:**<br>Ideally, all pupils will be achieving automaticity in all areas of the curriculum by the time they leave primary school. However, the carefully considered and sequenced curriculum should also provide opportunities for identifying and closing gaps in learning. This reactive aspect of the curriculum should seek out, rather than accidentally expose, such gaps in learning. | **Pupils are given extra time in lessons to rehearse core facts and methods**<br>Pupils benefit from extra rehearsal, but teachers need to assure themselves that what is being missed will not cause further problems down the line. For example, pupils rehearsing number bonds during a lesson where most of their peers are learning column addition means that these pupils will not be able to access learning later on. Instead, schools must work creatively, using available technology and, for example, parent support, to provide catch up that does not impact on learning during core maths lessons.<br>**Homework is set on topics where students need additional consolidation, but is not checked**<br>Homework can be effective method to consolidate and close gaps in learning. However, in the absence of additional/coupled systems for checking that homework is accurate and completed to a high enough standard, there is a risk that school leaders and teachers cannot assure themselves that gaps in learning are being closed. Further, there may even be a risk that pupils rehearse misconceptions and errors. Teachers should ensure that the nature of pupils' homework is in a format that both pupils can complete and that parents can help with. |
| 9) Memory: How well do plans prepare pupils for end of key stage tests?<br>**School friendly questions:** | **Proficiency in mathematics**<br>Although pupils will need to be familiarised with the test format, it is the overall quality, breadth and ambition of the curriculum underpinned by consideration of coverage, content, structure and sequencing that is the key to preparedness, so | **The school's policy is for year 5 pupils to sit KS2 'SATs' past papers at the beginning and throughout year 5 and year 6 in order to inform groupings, interventions and progress data**<br>Past papers use complex tasks and are therefore not very useful for diagnostic purposes. Further, relying on |

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| How do your plans ensure that pupils can *recognise* the facts, methods and strategies each test question requires for successful completion?<br><br>How do your plans ensure that pupils are equipped with adequate test technique? | that pupils have enough knowledge to approach any and all test questions with confidence and success. Working through every KS2 SATs question will only limit pupils to familiarity with a sampling of the content aligned test specifications. | past paper analysis limits consideration of links within and between concepts that would otherwise be defined and allocated to curriculum plans. For struggling pupils, repeated testing of unfamiliar content is not likely to foster motivation and may cause pupils to see the purpose of learning mathematics as being purely for passing tests. Teachers should be mindful that pupils do not go through their school experience accumulating misconceptions and gaps in learning as these are more difficult to deal with via end of key stage assessments for interventions. |
| 10) SEND: How well do plans anticipate and respond to the needs of different groups of pupils?<br><br>**School friendly questions:**<br><br>How do your plans ensure that pupils with SEND can learn and remember key facts, methods and strategies that they can take with them to their next stage of learning and in life?<br><br>How do your plans ensure that pupils aren't kept waiting for readiness, but instead given what is needed to forge ahead? | **A curriculum that engineers success from the start ensures that more pupils are able to keep up. The curriculum should reduce need over time, rather than magnify it**<br><br>**Identifying the causes of later difficulties:**<br><br>Successful identification and closing of gaps should not only be thorough and systematic, but there should also be a commitment to identify the likely ***cause*** of the gaps opening in the first place: this may be due to suspected or diagnosed disability, but it is also just as likely to be due to lack of exposure to core knowledge, including opportunities for overlearning. Viewing differentiation as *the* solution for mixed-ability classes may divert teachers' attention from identifying, remediating and preventing gaps in components, as well as hold groups of pupils back unnecessarily.<br><br>Teachers should extend pupils who have the requisite prior knowledge to think about concepts | **The school has a policy of always differentiating at least three ways**<br><br>This is may drive increasing workload as well as lead to a mechanistic approach to differentiation that does not start from a consideration of whether different tasks are needed and actually address pupils' specific needs.. Teachers should be given the discretion to judge when differentiation is and isn't needed and how it should be used, rather than applying blanket rules. Compulsory differentiation risks embedding low expectations as a group of pupils, over time, infer that they will always receive less ambitious work to complete and do not learn the content components which woud allow them to attempt more complex tasks.<br><br>**The school differentiates using 'all, most, some' or similar differentiated activities**<br><br>See above. Differentiation may be necessary but when identifying reasons for variation in pupils' capacity to understand a mathematical concept, teachers must consider differences in prior knowledge of necessary |

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| | in more complex ways. This requires that the teacher has a good grasp of mathematical ideas.<br><br>**Deciding whether to wait, or to whether to act:**<br><br>There should be no automatic assumption that pupils identified as having SEN cannot achieve, but curricular approaches should instead look for ways to engineer success. A combination of proactive, reactive and accommodating approaches should foster positive intellectual development in the pupil, rather than automatically change the environment around the pupils. The latter risks causing maladaptation in the pupil, as well as entrenching or reversing progression in learning. Teachers should be mindful that mathematical knowledge is biologically primary knowledge, and that development of intellect in this area will not happen of its own volition.<br><br>**Where mathematics is taught in sets (including fixed tables or groups)**<br><br>Teachers must have an awareness of how the sets they teach differ from each other and explicit appreciation of their differing curricular needs. High expectations for the lowest sets should not compromise the requirement for sufficient over-learning to ensure automaticity in each topic before progressing to more complex material.<br><br>Where mathematics is taught in mixed-ability classes,teachers should maximise opportunities | components. Teachers should be aware that to avoid gaps widening some pupils need to be taught and gain automaticity in the prior components that allows others in the class to attempt more difficult examples. For example, pupils learning about equivalent fractions are held back by their limited facility with multiplication tables.<br><br>**The school provides scaffolding so that less able pupils can access the concept**<br><br>It is not acceptable if lower prior attainers are always given scaffolded work and not taught or provided with practice to gain appropriate understanding and automaticity in the related mathematical concepts and procedures. The aim should be for the eventual removal of the scaffolding.<br><br>**The school's policy is for pupils to be in mixed ability groups so that peer learning can take place**<br><br>Pupils who have gaps in learning or SEN, such as difficulties in processing information, benefit most from the teaching and guidance of experts in their field. Expecting struggling pupils to learn from peers risks struggling pupils maladapting to copying, accumulating misconceptions, experiencing frustration or adopting opt-out/cover strategies. |

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| | for pupils with SEND to obtain enough practise of the most useful facts and methods within lessons. | |
| 11) Early years: How well do plans ensure that children in the early years are secure in basic mathematical vocabulary as well as the early learning goals for<br>a) Number?<br>b) Numerical patterns?<br><br>**School friendly question:**<br>How do your plans help children who do not have as much knowledge of number when they start school?<br>How do your plans make sure that pupils who aren't as familiar with mathematical content, are introduced to valuable words, concepts and symbols in a clear, precise and systematic way?<br>How do you ensure that pupils learn the content needed for success in year 1? | **Ensuring pupils are equipped with useful language, facts and methods ready for year 1**<br>Teachers and leaders of pupils in Reception year groups should be aware that many aspects of mathematical language, facts and methods do not develop of their own accord, and that a proactive approach going beyond play-based and child-led approaches will be needed. Plans should ensure that all pupils are equipped with what is needed to be successful in year 1 and beyond.<br>Plans should aim to close the school entry gap in knowledge of number and mathematical concepts and vocabulary, giving attention to the at-risk, vulnerable and disadvantaged groups. | **Teachers use dynamic planning approaches, responding to pupils' interests, questions and stage of development in the moment**<br>Highly articulate, confident and knowledgeable younger children may find this approach is engaging and will then seek out new learning experiences under these conditions. However, teachers should bear in mind that for those pupils who do not have prior mathematical knowledge, social skills or spoken language proficiency, such approaches risk presenting additional barriers to learning that would otherwise be overcome by more proactive and systematic approaches to teaching and practise of component parts. Care should be taken to not immediately label avoidance or lack of engagement as indicative of unreadiness or potential disability, but as opportunities to reach out and seek ways to engineer success. |
| 12) Key stage 1: To what extent do the curriculum plans ensure that the appropriate subject | **Ensuring pupils are equipped with the most useful and important facts and methods needed for success in key stage 2** | **The school have extended play-based learning into KS1; the mathematics curriculum is** |

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| content for key stage 1 is identified? | It is reasonable to expect that teachers of older primary pupils assume some prior knowledge of key facts and methods when they plan units of learning for pupils in their classes. Teachers and leaders of pupils in younger year groups need to ensure that pupils are helped to learn what is most valuable and forward facing in terms of being able to access the year 3 curriculum onwards. Part of this curricular deliberation should include attending to useful and efficient methods that can be used in more complex mathematics, so that working memory is not compromised. | **delivered through enabling, 'maths-rich' environments**<br><br>Children who do not bring in early knowledge of number from home, or who have significant gaps in knowledge, will struggle to access the intended curriculum unless action is taken to avoid this scenario. At-risk groups of pupils, in the absence of systematic approaches to instruction, rehearsal and assessment, could fall further behind and leaders should satisfy themselves that the curriculum enables at-risk groups to learn the necessary content that some, perhaps from more advantaged groups, may have learned. Teachers should also ensure that pupils do not acquire a misconception that hard work should only follow interest, as this may impact on behaviour for learning when pupils enter KS2. |

## 2. The extent to which teaching supports the goals of the mathematics curriculum

| **Focused questions to ask the evidence** | **Outline of potentially stronger practice in terms of intent, implementation and impact**<br>NB: answers will take many forms. Below are common findings to look out for | **Outline of weaker practice in terms of intent, implementation and impact**<br>NB: answers will take many forms. Below are common findings to look out for |
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| 13) What is the rationale for the teaching approaches chosen (is it primarily to achieve the curriculum intent)?<br><br>**School friendly questions:**<br>How do you match teaching approaches to the content being learned?<br>How do you choose the teaching approaches that give the most clarity?<br>How do pupils experience systematic teaching approaches in this school? | **Systematic instructional approaches: content that is given to pupils should be clear, concise, ordered and built upon pupils' prior knowlege**<br>This can take different forms: presentation, explanations, even storytelling, singing and recitation, for example. After explaining a concept, procedure or strategy, teachers can use questioning to reinforce, check for understanding and facilitate immediate rehearsal. The interaction between teacher and pupils should avoid overload by requiring that pupils make specific *intended* connections, drawing attention to what is important and what pupils need to remember, generalise and apply to other contexts. Teachers should also use and explain a high standard of mathematical vocabulary and accurate notation.<br>**Representations:**<br>Pupils should be given representations that expose the structure of a concept in ways that make sense to pupils. These can be in the form of diagrams, or even involve physical apparatus (if appropriate). Representations or analogies should be familiar, consistent and reinforced throughout a sequence of learning rather than spontaneously and inconsistently changed, so that pupils are able to see the substance of a question as opposed to | **The teacher's instruction and questioning causes pupils to guess or regularly make unintended connections**<br>Prior thought must be given to the use of questioning within instruction, where the expectation is that pupils will make the intended links for themselves. Teachers must assure themselves that pupils already have the prior knowledge with which to make links to, lest pupils resort to guessing, trial and error, or developing the habit of finishing the teacher's sentences.<br>**Teachers provide tasks aimed at developing new 'mathematical understanding' without a clear specific curricular intent**<br>Understanding is a continuum. Therefore, teachers should have clarity around *the degree of understanding* they intend pupils to gain of a specific curricular goal. At any one point in a pupil's journey through school, achieving understanding can be taken to mean acquiring a solid enough understanding of the mathematics that's been taught to enable pupils to move on to more advanced material. |

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| | expending working memory on getting past the surface features of a new analogy or representation.<br><br>Drawing pupils' attention to the deep structure of problems/exercises through 'thinking out loud', and through the provision of similar worked examples, enables pupils to become more familiar with underlying principles before being expected to work independently. | **"We ensure pupils can always explain their reasoning for the answers they provide"**<br><br>Pupils are not necessarily capable of articulating knowledge that they understand and are capable of using to learn more complex tasks. Teachers should consider the opportunity cost of requiring explanations, especially as this ability may be more easily developed as automaticity develops.<br><br>**There are regular mathematics tests**<br><br>This does not answer the question without some indication of how this data is used. The results of testing should influence the teaching a student receives in the future. For example, it might affect planning or any setting, or interventions used by the school.<br><br>**The school uses a mixture of informal means such as questioning and more formal testing**<br><br>Lesson by lesson, teachers should rely on the feedback via questioning and pupil responses to activities. More formal assessments may be less frequent but are necessary given the limits of teacher assessment. |
| 14) How do teachers ensure that key content is remembered long term? | **Systematic rehearsal approaches are methodical, linked to recently taught content and enabling links to be made in the pupils' minds. Pupils should have the confidence that comes from knowing what to do and being successful**<br><br>**Emphasis on memorisation of core declarative knowledge such as multiplication tables:** | **Pupils do not know what to do and many are reliant on adult help**<br><br>This may indicate something about the curriculum, but it may also indicate that the tasks themselves require too much of the pupils in terms of both remembering what to do as well as recall and use previously learned content. This is more likely if the task is not related to the examples teachers used |

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| **School friendly questions:**<br>What do teachers do (in this school) to make sure that all pupils can remember and repeat the words and concepts being taught?<br>What do teachers do to make sure that pupils obtain enough practise in the classroom?<br>What do teachers do and say that emphasises key facts, methods and strategies? | Automatic recall of number facts is important to all learning in mathematics to avoid cognitive overload and enable pupils to focus on concepts. The school should have a structured plan, identifying which facts are to be learned in each year by those pupils who have automatised the previous years' learning. These should include addition and subtraction facts, multiplication and division facts and other key facts such as common equivalents e.g. 1/4 is equivalent to 25% and 0.25.<br>**Purposeful practise:**<br>Pupils' understanding may also be developed through purposeful practise of exercises where the procedure is known, but the answers are not. When working through sufficient exerices, pupils gain a greater familiarity with the underlying principles of procedures, as well as familiarity with patterns of number families. Teachers may choose to make the patterns and underlying principles even more apparent by adding an element of intelligent variation to the sets of questions.<br>Pupils' understanding may also be developed through applying recently learned procedures to linked problem solving where problems with a similar deep structure are grouped together. This also enables pupils are to see and learn the commonalities and differences of the problem type, so that they acquire the ability to see past the surface features of linked, novel problems and to the deep structure.<br>Understanding that is developed through purposeful practise of exercises and groups of similar problems is not readily observable, as it takes place within the pupils' mind. | during input. The risk is that in figuring out what to do, pupils will have sacrificed thinking time that could have been devoted to content. Furthermore, pupils who are overly reliant on additional adult support may be lacking in knowledge of component parts that may have made otherwise clear the reasons why and how an activity works.<br>**The school's policy is for most calculations work to be completed on white boards**<br>While this may allow more flexibility in terms of moving through content, enabling pupils to demonstrate understanding and recall, and offers pupils the chance to experiment with solving problems without the pressure of showing their working, the risk is that pupils do not commit procedures for systematic ordering of calculations (presentation) to long-term memory. Further, they risk missing out on opportunities to see their own progress and familiarise themselves with prior content through being able to look through their own books.<br>**Pupils are enjoying theactivities, but aren't attending to content**This could indicate that pupils are enjoying the feeling of success that comes with knowing more and gaining further insights into mathematics, or it could indicate a liking of the activity itself. Teachers should be mindful that activities designed to be enjoyable do not cause pupils to focus on and remember the activity, rather than the maths. |

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| | **Spaced retrieval practice:**<br><br>Learning is not linear. Pupils unlearn as well as learn. Teachers should appreciate this and continually monitor for advances in mathematical knowledge, as well as regression when content might be forgotten. A systematic approach to retrieving content helps to stop the forgetting of content over time, and helps pupils to make new links within and between content. Regular quizzes and tests can form an important part of a system for regular retrieval.<br><br>**Bookwork:**<br><br>If exercise books contain lots of exercises or other practise using previously taught facts, concepts and procedures, then this should not be criticised in the absence of further investigation. *If there is evidence of a lack of progress through the curriculum,* then the overall challenge of the curriculum plan should be the target of criticism, and not repeated practice to build automaticity.<br><br>**Questioning:**<br><br>Teachers may choose to use questioning that refers to or helps pupils to recall previously learned, but related, content, or they could consider the use of familiar and powerful analogies and mnemonics. Ensuring that pupils can use recently learned content in different ways is particularly powerful when pupils recall in unison. Teachers should take care to ensure that pupils avoid the temptations of copying.<br><br>**Pupils' attendance to content:**<br><br>Pupils need to think about content enough for there to be a change in long-term memory. The teacher may consider the benchmarks for practice that enable the majority to learn | |

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| | new content. Homework could also help pupils to familiarise themselves with previously taught content. However, all pupils benefit from opportunities to overlearn material covered in lessons.<br>**Working memory:**<br>Activities should not overload working memory or cause distraction through requiring choices about resources, seating and activities. | |
| 15) How effectively does the classroom environment support pupils' learning?<br><br>**School friendly questions:**<br>How do teachers make sure that pupils have enough undisturbed time and space to consider new content without being overwhelmed or distracted? | **Pupils need to see, hear, think and talk about mathematics for themselves**<br>**Hearing and understanding content:**<br>**Clarity of written workings:**<br>It is important that pupils have the best chance of seeing the deeper structure of problems, exercises and caculations through being given opportunities to learn how to lay out their work in a way that is clear and precise. For those pupils who struggle with presentation, for example with number formation, there should be procedures in place to ensure they can re-learn new presentation habits. This may help them to see patterns of number more clearly (such as place value),. Their work needs to clear enough to be followed by themselves and others. Through taking care with presentation, pupils are also more likely to develop pride in their work, as well as to be able to spot errors as they arise.<br>**Clarity of thought:**<br>When teaching complex concepts or procedures, it is generally necessary to break them down into logical steps and ensure that all pupils understand and have automaticity in each step before moving on. Teachers should be assured | **The classroom is noisy and there is a high amount of pupil movement**<br>Many pupils thrive amidst a 'learning buzz'. However, teachers should also consider the need of pupils who speak English as an additional language, those with learning disabilities and those who struggle to concentrate. The resulting auditory and visual information received by at-risk pupils would overload working memory and cause pupils to miss key parts of teacher explanations, lose their train of thought when trying to calculate, or cause them to make mistakes that may result in errors and misconceptions. Teachers should be mindful of modern workplace misconceptions potentially influencing pupil experiences – experts in the field of mathematics and those who use mathematics for a living tend to work in very quiet, low-distraction and interaction environments.<br>**Pupils are not clearly informed when their answers or methods are wrong**<br>It might seem kind to refrain from clearly pointing out where errors have been made, but pupils benefit from honesty and the chance to learn from their |

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| | that pupils know what to do before being asked to work independently, lest they recourse to guessing, copying or becoming reliant on peer/adult support.<br><br>Learning materials should be free of distraction, such as the requirement to make many choices about equipment, and ensure that information is integrated in as optimal a manner as possible (e.g. information on diagrams is integrated and not on separate pages).<br><br>If transitions are a key feature of the lesson design, teachers could consider whether a transition could be used as an opportunity to rehearse content, or whether it is best for pupils to remain thinking about what has just been taught to ensure greater success during independent work.<br><br>**Clear and concise language:**<br><br>Mathematical vocabulary and notation should never be shied away from. It should be modelled by the teacher and expected from the pupils, who should be given opportunity to use it regularly in written and oral work until it becomes as fluent as procedures. Pupils have more chance of successful understanding of teachers' explanations if they can see what the teacher is pointing to and 'see' what the teacher is saying (this is especially the case for pupils with EAL who need to 'see' the articulation and enunciation of words).<br><br>Careful questioning may help pupils to hone their reasoning, particularly if the teacher gives them more accurate language to use when they are explaining their thinking. The use of questioning can also be an opportunity for all pupils to take part, regardless of who is chosen: if pupils know that they are all equally likely to be called upon, then they will all think about a suitable answer to a | mistakes. If pupils are consistently making mistakes, then teachers need to consider whether their mistakes are the result of gaps in learning or lack of systematic rehearsal of the basics, rather than always indicative of a need for more explanations.<br><br>**The school requires that pupils always work in groups**<br><br>There are times when group work can be beneficial. However, for content to be committed to long-term memory, pupils need to be given time to think about and make sense of core content by themselves. Further, when further along in a sequence of learning and pupils are required to solve problems requiring the holding of a train of thought, disturbance by voices and noises will disrupt these trains of thought. This would prevent pupils from solving problems and stop them from acquiring valuable conditional knowledge. Teachers should ensure that all pupils can contribute and get something out of working in a group, if groupwork is to be used.<br><br>**The classroom environment is colourful and vibrant, and pupils have access to multiple facts, vocabulary, concepts, symbols and explanations that are on display around the room**<br><br>Teachers should strike a balance between judicious use of information to display, and the risk of distraction and cognitive overload for at-risk groups of pupils. A visually 'busy' classroom environment is not going to lead to enhanced learning, and in some cases can prevent learning from happening. For some |

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| | question. Teachers should consider whether questions are reliant upon pupils' wider knowledge brought in from home. | groups of pupils, visual overload can be quite distressing. |
| 16) How effectively are barriers to learning overcome?<br><br>**School friendly question:**<br>How do teachers get the best out of children who would normally be expected to struggle?<br>How do teachers make sure that classroom support doesn't create further barriers later? | **Prevention of barriers to learning is better than circumvention of barriers to learning**<br>Approaches that seek to prevent rather than circumvent barriers to learning should be preferred. For example, pupils with poor decoding skills should be taught to decode key vocabulary and the school should be aware of the impact of inadequate command of spoken or written English on mathematics performance.<br>Autism – removing the barrier of social skills needed to access content by ensuring group work does not predominate lesson structure. Periods of silent working coupled to deployment of known facts, methods and strategies to reduce anxiety around what to do. Harnessing the powers of systematic pattern seeking through intelligent variation to allow pupils to uncover links between knowledge, rather than require their discovery through random trial and error. Use of familiar routines to minimise anxiety about what is happening next. Testing for success rather than testing of content that has not yet been taught, learned or applied in wider contexts.<br>Global delay – processing speed accounted for/mitigated, with provision of additional practise to help learn core knowledge to automaticity.<br>Dyslexia – taking care with explanations and vocabulary, ensuring that pupils have enough phonics and vocabulary knowledge to decode and understand questions.<br>Dyscalculia – as for global delay, particular attention to avoid all possible causes of confusion and systematic assessment for gaps in core knowledge. Provision of | **"We liaise with the SENDco to provide support and/or a relevant language specialist"**<br>Teachers should consider the additional cognitive load imposed by poor English language skills and provide oral work and/or translations to minimise it.<br>**The school's policy is for identified pupils to be allowed to opt-out of maths activities**<br>It may be beneficial to allow opt-out during a moment of distress. However, habitual or organisational permission to opt out may risk a pupil narrowing their own mathematics curriculum, accumulating gaps in knowledge and increasing the likelihood of struggle later. Teachers should consider that anxiety and distress due to the pupil experiencing (or anticipating) confusion could otherwise be overcome by closing gaps in knowledge so that pupils can take part with more confidence. |

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| | overlearning opportunities so that core facts are learned to automaticity. | |
| 17) How effectively are resources used support teaching and learning of mathematics?<br><br>**School friendly questions:**<br>How do teachers use resources to aid pupils' understanding and how do they ensure that those same resources don't become a crutch?<br>How do teachers make sure that pupils are thinking about content, rather than distracted by the shapes and colours of manipulatives, for example?<br>How do teachers facilitate pupils' accuracy, precision and logic in their written work? | **Resources should make content clear as well as provide a way for pupils to recap, consolidate and feel proud of their success**<br>**Concrete resources/manipulatives:**<br>The ideal use of concrete resources is as an aid to reveal mathematical structure and develop understanding of the concept being taught. The resource should therefore be selected in relation to the aspect of the concept that is being taught, as well as be familiar or consistent in its use.<br>**Textbooks:**<br>Textbooks can be very effective in providing a set of quality coherent materials to support the teaching and learning of mathematics. It is important however that they are used intelligently. Teachers should study the textbook, ensuring that they understand the mathematics to be taught, resources and images used and why they have been selected. The curricular intent and not the textbook should lead planning and the teacher should be responsive to errors and misconceptions*.<br>**Exercise books:**<br>Exercise books can be a resource for revision as well as a place for pupils' work, but a resource for revision. Teachers should be mindful that requiring pupils to do most of their work on white boards will remove further opportunities to revise the steps in calculations. Excessive whiteboard work also risks preventing the developing of systematic calculations and pride in work. | **The school use the "Concrete Pictorial Abstract approach (CPA)" in every lesson**<br>Such approaches should not leave pupils for too long at the 'concrete' stage, hampering their progression.<br>One of the misconceptions of the CPA approach is that it is followed in a linear order. First, we work in the concrete, then the pictorial and then move on to the abstract. Whilst this is sometimes the case, the real power comes when the three forms are seen together or in short succession and understanding of the abstract is connected and supported by concrete and pictorial images. The end goal should always be for children to become fluent in working with abstract symbols and representations. The concrete and pictorial are means to achieve this.<br>"**We encourage pupils in all year groups to select from a wide variety of manipulatives to aid their understanding during independent activities"**<br>The process of selection can be distracting, and if pupils are using manipulatives as a matter of course, then this would indicate lack of learning of core knowledge to automaticity. Pupils should not be reliant on manipulatives and reliance should not be encouraged.<br>**"We use textbooks to guide the teaching of mathematics"** |

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| | *Unless in being used in other countries where national textbooks match and exactly follow a highly sequenced national curriculum. | There is no virtue on creating exercises which already exist in textbooks. School policies on resource creation should not drive unnecessary workload.<br><br>**Pupils' independent experience of the intended curriculum is only ever via the use of loose worksheets or photocopied pages of textbooks**<br><br>Pupils may benefit from being able to handle and refer to textbooks, both to help them remember and recall previously taught content (particularly worked examples and explanatory diagrams) as well as gain a sense of progression and achievement. |

## 3. The effectiveness of assessment

| Focused questions to ask the evidence | Outline of potentially stronger practice in terms of intent, implementation and impact<br><br>NB: answers will take many forms. Below are common findings to look out for | Outline of weaker practice in terms of intent, implementation and impact<br><br>NB: answers will take many forms. Below are common findings to look out for |
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| 18) a) What is the rationale for which content is chosen to be assessed and<br><br>b) how is that content assessed?<br><br>c) how is any assessment used to | **Proxies for learning:**<br><br>Teachers can decide the appropriate proxies for learning (e.g. ability to apply the formula for the area of a square in a multistep question) and then use these in informal tests.<br><br>Teachers should be aware that some proxies for understanding are not as reliable, e.g. pupil explanations. Lack of explanations may indicate the absence of vocabulary/language that would clarify, or it may indicate early learning of a procedure that has yet | **The school sets tests at the beginning and end of each topic to demonstrate progress**<br><br>Using assessment to 'demonstrate progress' rather than genuinely inform teaching is unhelpful.<br><br>If 'progress' does not include understanding and automaticity in identified facts, concepts and procedures to allow further progress, then it is not adequate. Teachers should be mindful of what pupils learn when the beginning of any topic starts with a test of content that has not been taught. Inspectors |

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| inform future curriculum design?<br><br>**School friendly questions**<br>Which content do you assess and why?<br>What assessment approaches do you use?<br>How do you use results?<br>How are parents kept informed of their children's assessment results and do they know what is being assessed at the time? | to reveal insightful connections within and between the knowledge in the mind.<br>**Different forms of assessment are appropriate for different purposes:**<br>Assessment can be:<br>AS learning (memory building)<br>FOR learning (formative assessment to identify gaps)<br>OF learning (summative assessment to create data) | can use activities to check if all pupils have understood and reached automaticity in the intended curricular goals.<br>**The school uses a tracking system to map the progress of each pupil**<br>Tracking systems can be helpful. However, combination of 'just in time' teaching and 'one and done' approaches to observing pupils meeting criteria may drive artificial acceleration through the curriculum and an overall neglect of the need to help pupils retain content in long-term memory.<br>**"We use self-review and peer-review"**<br>Young pupils are the least likely to be aware of their own and peers' misconceptions, errors or gaps in learning. When young pupils are expected to use self and peer review, the risk is that 'gut instinct' and overall feelings about enjoyment of an activity become a default mechanism for assessment. Teachers should ensure that self and peer review is informed by readily available and accurate comparator information.<br>**The school avoids tests as they make pupils anxious and take time from teaching**<br>Tests are not stressful when pupils are adequately prepared for and used to them. Tests are also useful means for gathering information quickly as well as one of the most effective means of consolidating learning. If pupils are anxious, teachers should aim to engineer success so that tests are approached with confidence and not fear. |

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| | | **Parents do not know how well their child is doing**<br>Parents may wish to be given clear, quantitative information about where their child is relative to the national average, as well as what the benchmarks are for year group expectations, so that they can support their child to prepare for and do well in key tests of automaticity and composite skills. This is preferable to more generic statements about progress that do not alert parents to the fact that their child's *rate* of progress is such that he/she is unlikely to meet end of key stage expectations and thus potentially be disadvantaged in Year 7. |
| 19) How is automaticity assessed?<br><br>**School friendly question:**<br>How do you know whether pupils have obtained speed and accuracy in recall of facts and methods?<br>Have you decided on any standards in speed and accuracy of recall of maths facts?<br>How do you assess younger pupils' recall of number bonds? | **Systems of frequent, low-stakes (timed) testing of core facts benefit pupils' learning and provide diagnostic information for teachers**<br>Core knowledge needs to be secure in memory and readily recalled if they are to be successfully applied in more complex tasks. Frequent, low-stakes and timed testing of declarative knowledge (not problem solving, a composite skill) is useful in this context.<br>Assessments of automaticity must have carefully defined benchmarks which measure accuracy and speed of recall while ruling out a reliance on derivation (eg. the use of skip counting to complete times tables tests).<br>Assessments of automaticity can also be extended to testing of procedural knowledge in the four operations, providing teachers are confident that pupils have learned the procedures beforehand. | **The teacher makes a judgement on this using pupil work and in class feedback**<br>Exercise books and performance in class are not necessarily reliable indicators of automaticity.<br>**Pupils struggle when given recap activities**<br>Automaticity comes with overlearning. Therefore if pupils need explanation and support with recap activities, it suggests content is not being learned to automaticity as this requires practising *beyond* accuracy. There should be evidence of extended recall in which pupils can work swiftly and obtain correct answers. |

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| How do you assess automaticity in the maths facts required to access year 7 content e.g. times tables, prime, square and cube numbers? | | |
| 20) What types of assessment verify end of key stage test preparedness?<br>**School friendly question:**<br>How do ordinary tests of recall build children's confidence for the final performance? | **Pupils should have enough proficiency in mathematics to access learning in year 7, rather than enough for meeting end of key stage test expectations**<br>Pupils benefit from being familiarised with test formats through attempting some past papers, but the main goal should be for pupils to understand and have automaticity in the facts, concepts and procedures. | **The school sets regular past papers**<br>Experiencing the final performance is not the best way to learn what is needed for the final performance. KS2 statutory tests only sample pupils' learning, and the benchmark for meeting expectations, which corresponds to knowing around 50% of KS2 curriculum content, is not necessarily a benchmark that will ensure success in year 7. Teachers need to ensure that pupils are proficient in all core knowledge. |

## 4. The extent to which there is a climate of high subject expectations where a love of mathematics can flourish

NB: This focus may well help explain the success of some schools, but a lack of evidence for 'climate where a love of the subject could flourish' could NOT reasonably be deployed to explain weakness given the challenge of identifying this during inspection.

| Focused questions to ask the evidence | Outline of potentially stronger practice in terms of intent, implementation and impact<br>NB: answers will take many forms. Below are common findings to look out for | Outline of weaker practice in terms of intent, implementation and impact<br>NB: answers will take many forms. Below are common findings to look out for |
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| 21) What is the profile/status of mathematics in school | **The subject is seen as both facilitating study in other subjects as well as being worthy of study because it is interesting** | **The school discourages the use of rewards so that pupils develop intrinsic motivation instead** |

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| compared to other subjects?<br><br>**School friendly questions:**<br>Do pupils find mathematics interesting?<br>Do they know it's important?<br>Is proficiency celebrated? | Schools should take care that the high priority of mathematics does not lead to ever-increasing lesson duration and frequency. Such an approach risks lowering efficacy through pupil fatigue, as well as promoting inefficient classroom pedagogy.<br>**Rewarding of success and hard work:**<br>Regularly giving pupils real knowledge of their own ongoing success will help them to see, for themselves, the results of their hard work.<br>Defined benchmarks and metrics for success should be coupled to regular, quantifiable, objective information that is given to pupils and parents. This is more powerful than generic messages of praise and progress.<br>Pupils should have their success celebrated. | It is a fallacy to believe that extrinsic motivation in the form of rewards, praise and competition will stop pupils from developing intrinsic motivation. Pupils' love of the subject and further motivation to learn develop from *success* in and knowledge of the subject. Therefore, teachers should firstly seek to engineer genuine success. For novices who struggle, particularly the youngest pupils, extrinsic rewards may provide the initial impetus that allows success to flourish, and pupils' interest develops more from there.<br>**Pupils have formed a poor view of the subject**<br>Adult attitudes can affect children's attitudes, even when the adults may think they are careful. Younger children are particularly vulnerable to overhearing and internalising messages about disliking mathematics, or attributing low proficiency due to a familial trait. Teachers and support staff can help children to overcome this kind of messaging by reiterating the power of practise, as well as by ensuring pupils have opportunities to learn and feel successful. |
| 22) How do staff ensure there are high expectations for all pupils?<br><br>**School friendly questions:**<br>Do pupils know what 'good at maths' looks like in order to aspire to that standard? | **Motivation as the outcome of success:**<br>Teachers should be aware that motivation and love of a subject is predicated on success in that subject. The curriculum and associated pedagogy should engineer that success rather than offer opportunities, rather than wait for its development.<br>Schools should be explicit that learning is what counts. Having explicit goals for each topic gives older pupils an achievable goal that will motivate them to work hard to learn the next topic. | **The school makes sure that every mathematics lesson is fun and relevant**<br>No teacher should aim to be boring but schools should also aim for pupils to be successful and then see the subject as interesting in its own right.<br>If the subject is made interesting with novel activities, then pupils might only remember the novelty and not the learning intention.<br>Teachers as role models can speak of their love of mathematics and why mathematics is, in its own right, a worthwhile pursuit that does not require a practical |

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| Do pupils know that profiency, motivation and love of the subject are the fruits of hard work, focus and attention to accuracy, logic and detail?<br><br>Do parents know what 'good at maths' looks like in each year group so that they can help their pupils at home?<br><br>Are pupils held accountable if they choose lower expectations of themselves?<br><br>Do pupils know when and where the errors are in their calculations? | **Pupils aspire and are held accountable:**<br><br>Steady progress in achieving conceptual and procedural fluency not only enables pupils to achieve, but it also provides the motivation and builds strong habits of application. Testing - if these assessments are set at the right level, the success rate will be high and pupils will look forward to them. Pupils should know that hard work and focus are what count.<br><br>Pupils benefit from encounters with the awe and wonder of mathematics as demonstrated through teachers' subject knowledge and expertise. | application to make it meaningful. Pupils can understand that learning mathematics, like art or poetry, helps them to grow as a human being.<br><br>**"We don't use competition or testing for fear of 'mathematics anxiety'"**<br><br>Pupils can be highly motivated by tests, quizzes and competitions when they offer a chance to demonstrate how much they have learned. Competitions are only demotivating when pupils are poorly prepared.<br><br>**Pupils are only striving for minimum standards**<br><br>Pupils risk internalising a poor work ethic if they have been given a low bar to aim for, or if they have internalised that proficiency (or lack, thereof) in mathematics is an unalterable character or genetic trait. This can happen if teachers give them specific goals to merely 'pass' tests, or if they are on tables or in streamed classes where they never see what the 'gold standard' looks like or entails. |
| 23) How do you enrich the curriculum beyond classroom learning?<br><br>**School friendly questions:**<br><br>Is there something extra on offer for the number lovers?<br><br>Do proficient mathematicians know what | **Trips**<br><br>Pupils can also benefit from knowing where mathematics can take them, both in terms of career and in terms of understanding and taking part in conversations that require mathematical literacy. Pupils may also benefit from knowing about how mathematics underpins modern technology, advanced engineering and scientific discovery.<br><br>**Clubs and competitions:**<br><br>The curriculum should not only make provision for enrichment opportunities and further challenges, but | **Budgets prioritise interventions, workshops and catch-up support, leaving no provision for pupils who show high potential and interest**<br><br>While funds such as pupil premium can be directed to supporting disadvantaged pupils because they are, statistically speaking, at risk of falling behind or not meeting expectations, leaders should be aware that the overall purpose of raising attainment for pupils who meet the criteria for pupil premium funding is for all pupils who attract said funding, including those who meet minmum expectations for the purpose of school |

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| to look forward to in the future? | ensure that pupils and parents are aware of the pathways to its access. | accountability and yet show potential to achieve highly in the field of mathematics. |

## 5. The quality of systems and support for staff development

| **Focused questions to ask the evidence** | **Outline of potentially stronger practice in terms of intent, implementation and impact**<br><br>NB: answers will take many forms. Below are common findings to look out for | **Outline of weaker practice in terms of intent, implementation and impact**<br><br>NB: answers will take many forms. Below are common findings to look out for |
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| 24) What do the strengths and weaknesses already identified indicate about the school's capacity to design and implement effective mathematics education?<br><br>**School friendly questions:**<br><br>What are the strengths in your provision of mathematics education?<br><br>What are the potential issues?<br><br>How do you ascertain whether potential issues | **The school has sought out schemes of learning, equipment and resources that are proven to be effective and equitable. Project management of implementation ensures that component parts (such as teacher CPD) are embedded at each stage**<br><br>Such an approach is likely to yield positive results that build over time, extending beyond increased attainment and automaticity to increased motivation and love of the subject. Within this paradigm, pupils will have a real sense that they are proficient and parents will have clarity and respect for the kind of mathematics education provided by the school. | **The school is overly reliant on parental help, homework, tutoring and interventions to bridge the gap between intent and impact**<br><br>This approach can work in the short run, but ultimately neglects the root cause of systemic issues in curriculum planning and implementation. An analytical approach would be needed to ascertain whether the problem lies with intent or implementation; there should be no automatic assumption that an entirely new curriculum is needed. Conversely, care should be taken when choosing curricula based on a predeliction for all things 'modern' and 'fun' rather than evidence-based approaches.<br><br>**The school considers its higher than average results indicate that all is well**<br><br>It should not be assumed that favourable results and higher than average attainment represent the zenith of curricula and pedagogical approaches. This is partly because the current rates of attainment still permit |

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| originate in the curriculum plans, teaching approaches or pupil practice?<br><br>How do you promote stability and gradual change for the better, as opposed to overly reactive approaches?<br><br>How do you limit the need for resource-consuming interventions later on? | | roughly one in five pupils to transition to secondary without a nominal proficiency in mathematics.<br><br>**Teachers are required to adopt inefficient and ineffective pedagogies**<br><br>A subject lead that insists that pupils must 'find out for themselves' to aid their 'understanding' risks lessons being predominated by exploration with the role of intended content relegated to momentary consideration at the end of the lesson. For example, if the intention is to learn how to find a missing angle in a geometry problem, spending the majority of the lesson devoting thought and attention to 'finding out' will lead to memories of exploration rather than the useful conditional knowledge. Further, if pupils are unsuccessful, their memories will also be linked to feelings of frustration and failure. |
| 25) What is the quality of the subject level processes, including of curriculum construction, debate and renewal? How does the subject lead evaluate the strengths and areas for development (if there is an identified leader)? | **Internal processes:**<br><br>Curriculum development should be a normal part of schoolal activity and clearly evident in schoolal meeting discussions. There should be no imperative to 'reinvent the wheel' but all teachers, even non-specialists, should be encouraged to identify problems and if possible suggest solutions. The errors that pupils make should suggest areas where the scheme of work might be improved, and of course changes in exams may necessitate changes in the curriculum.<br><br>Ideally, both the granular detail and the 'bigger picture' of curriculum has not only been worked out, but is also able to respond flexibility to different knowledge profiles of rising year 7s. Ideally, the curriculum should become more stable over time. | **The school head ensures that teachers receive feedback on the curriculum as they teach**<br><br>The mechanisms for this feedback should be clear and the school should be able to give examples of how this has worked in practice. Particularly where the curriculum is designed by a MAT or other external body, are there functional mechanisms for teachers to discuss the curriculum and feedback on possible adjustments?<br><br>**The curriculum is in a state of flux; it is more reactive than proactive**<br><br>Curriculum design and implementation strategies that start with searching for and closing gaps, requiring pupils to repeat tests and then attend interventions, will consume human and physical resources, leaving |

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| **School friendly questions:**<br>Have you identified topic areas where pupils tend to struggle?<br>If so, how did you come to that conclusion and what short- and long-term processes can you use to remedy the situation? | **Liasing with other schools/outside agencies:**<br>This is good practice. Professional learning networks can be used to hear case knowledge from colleagues in other schools and from other external experts. Teachers can use school-supported networking opportunities effectively to articulate their own theories and to test their ideas through peer review. It is good practice if the school is supportive of these links. | little that can be deployed for proactive strategies. This is also not good for pupils in at-risk groups who internalise that they do not need to focus as much due to the likely provision of interventions.<br>**The school has 'rigorous monitoring systems' in place**<br>Systems should be focused on curriculum content and how pupils are acquiring content, rather than focusing on the surface features of what the teacher is doing, saying and using in the lesson. |
| 26) a) To what extent do requirements for consistency allow any necessary flexibility?<br>b) How is a consistent quality of teaching ensured from inexperienced, non-specialist and struggling staff?<br>**School friendly questions:**<br>How do you ensure that pupils receive a high-quality mathematics education, regardless of the relative | **Changing teachers:**<br>It is good for pupils to be taught by adults with different experiences, personalities, interests and subject specialisms. Each year with a different class teacher will add colour to the pupils' overall education. However, without coordination, the variation in instructional approaches and curriculum sequencing risks being too wide. Therefore, leaders need to have a system that ensures continuity and consistency for pupils' overall mathematics education, so that they do not face arbitrary barriers to learning.<br>**Engineering teaching success for NQTs and non-specialists:**<br>It is better to engineer success, rather than wait for and then mitigate failure. Provision of detailed schemes of learning, where the curriculum is annotated with pedagogical advice on how to teach each aspect, is a good idea. Inspectors would hope to | **There are no school wide protocols to ensure consistency**<br>Prioritising 'teacher autonomy' in the absence of checks and balances can also present further barriers to learning. For example, if a teacher prefers pupils to use repeated subtraction instead of teaching the use of short division, then later teachers may find pupils cannot access content such as converting fractions to decimals.*<br>*as well as factorising cubic polynomials at A level |

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| level of subject and pedagogic experience of their class teacher? | hear of more specific plans than claims of general 'support'. It is also a good sign if more experienced staff help resource lessons or are named persons to go to.<br><br>Beginning teachers also benefit from opportunities to observe experienced staff as part of their personal development. | |
| 27) How are all staff in the department encouraged to develop their subject knowledge and knowledge of how to teach that subject knowledge (pedagogical content knowledge)'?<br><br>**School friendly questions:**<br><br>Have you encouraged teachers to renew their knowledge of mathematics so that they know what pupils will be required to learn at secondary school?<br><br>How do you provide the opportunities for teachers to learn useful techniques and approaches that would | **Sharing knowledge about the teaching of mathematics:**<br><br>All school heads should have procedures in place to assess teachers' subject content and pedagogical knowledge. It should not be assumed that a mathematics degree will necessarily equip a teacher with adequate and up-to-date knowledge to teach every topic in the school's progression. The subject knowledge required to teach each topic should be specified and materials should be available for self-study. Ideally there should be a culture of discussing approaches to communicating mathematical ideas. Through collaborative practice, a strong school will challenge each other, refine approaches, implement known effective practices and evaluate their pedagogy against best evidence.<br><br>**A culture of knowledge acquisition:**<br><br>While the mathematics skills test for entry to the profession is set at roughly the equivalent of a C at GCSE, teachers benefit from aiming to improve their subject knowledge so that they can fully understand how the component parts of content learned in Reception year and throughout primary school | **The school don't have much time for this**<br><br>Heads of School should lead by example by explaining how they are using research to improve their own performance in the classroom. However, this will be meaningless if staff don't have the time or mental energy to read and reflect on their practice. Heads of school should understand that professional learning is key to improvement, and remove all unnecessary burdens from the team (e.g. administrative tasks that will not be used to inform and improve learning episodes) to allow opportunities for subject specific professional learning.<br><br>**The school provides training sessions for all teachers and sometimes sends teachers on external courses**<br><br>Although these two features have value, are they part of a coherent plan which has identified areas where staff need support and acted to remedy these issues? |

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| improve their teaching confidence? | influence and underpin what can and can't be learned at GCSE and A level. | |

## 6. The extent to which whole school policies affect the capacity for effective mathematics education

| **Focused questions to ask the evidence** | **Outline of potentially stronger practice in terms of intent, implementation and impact**<br>NB: answers will take many forms. Below are common findings to look out for | **Outline of weaker practice in terms of intent, implementation and impact**<br>NB: answers will take many forms. Below are common findings to look out for |
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| 28) What criteria are used to decide on the staffing priorities?<br>**School friendly questions:**<br>How do you ensure that all pupils get their fair share of staffing expertise? | **An equitable approach to timetabling**<br>**The subject lead has a large amount of subject knowledge and ensures that at-risk pupils receive teaching from the most highly qualified staff**<br>Such an approach is likely to be effective, preventing the need for increasing interventions and catch up over time. | **Inequitable approaches to timetabling: older year groups given priority resources at the expense of other pupil groups**<br>Balance must be struck between the need to identify and give older pupils the curriculum content they may have missed out on over time and the needs of younger pupils. Staff expertise and subject knowledge can be very effective when deployed for the education of younger and at-risk pupils, potentially avoiding the need for interventions later on. |
| 29) How do school-wide policies, e.g. marking or CPD, support your school's needs? | **School policies that should allow subject-specific flexibility**<br>Leaders should strike the balance between an organisation's need for professional consistency, where all stakeholders are commited to the same cause, and the need for what is best and most efficient for successful study in mathematics. Teachers should be able to to avoid | **The school's policy for pupils with SEMH is to provide a 'safe space' so that pupils who are struggling with their emotions in the lesson can opt out according to personal need**<br>Leaders should be aware that the consequences of this policy may be the prevention/disruption of learning for other pupils and inadvertent narrowing of curriculum |

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| **School friendly questions:**<br>Is marking manageable and do pupils know whether they've got things right or wrong?<br>How do you ensure a consistent approach to calculation and presentation? | unhelpful practices and unnecessary workload that may result from blanket policies that apply across an entire school.<br>**School policies that should foster uniformly high expectations of pupils:**<br>Regardless of the year, class or group that pupils belong to, all pupils and their class teachers benefit from school-wide policies that promote uniformly high expectations of bookwork quality, attention, behaviour, lesson preparedness (e.g. equipment) and homework completion.<br>**School policies on rewards and certificates of commendation should signal that intellectual success is as valuable as other areas of interest**<br>The celebration of mathematical proficiency born of hard work and attention to detail should be celebrated just as much as sporting prowess or artistic accomplishment. Ensuring fair access to rewards and commendations avoids pupils who are attaining well in mathematics from inferring that proficiency in the subject is accidental, natural or not as important as other subject areas. | for those pupils given the opportunity to opt out by choice. Such policies should be rigorously analysed to ensure that pupils who are provided with designated spaces and just-in-time mentoring don't end up with a habit of avoiding particular subjects.<br>**All teachers are required to follow a school policy which advocates daily marking with written comments, with extensive follow-up of pupils' responses to 'next steps' commentary**<br>Extensive written comments give little return on investment of time, not just because of the nature of the subject, but also because at-risk pupils are least likely to be able to respond to or even read teachers' comments.<br>Unhelpful policies can lead to superfluous comments that repeat the learning intention, such as:<br>"You have done very well on this piece of work and have shown your understanding of multiplication effectively through the drawing of arrays"<br>However, if a pupil has consistently made an error it would be good practice to see a modelled answer next to one error followed by a request to do the corrections. |
| 30) What are the priorities for discussions at management meetings with subject leaders? | **There is a joined-up, team approach to improving mathematics education**<br>Where all teachers and leaders are seen as equally instrumental in engineering success for young mathematicians, average attainment and therefore proficiency in the subject is likely to steadily build over time (and is less likely to fluctuate). Discussions within this | **Most of the responsibility for attainment (a proxy for proficiency) lies with the senior leader who is also the year 6 class teacher**<br>This approach is likely to lead to higher workload for the year 6 teacher as cohorts present with wider and wider ranges of attainment that must be catered for. Leaders should also think about the efficiency of such |

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| | environment not only concentrate on intelligent systems to carefully identify and close gaps, but also on what is necessary to prevent gaps and dysfluency from happening over time. This is more positive than relying on systems that look for gaps and then seek to assign blame. A culture of honest discussion, reflection and constructive feedback also helps teachers to feel valued and to want to improve their approaches to bringing the mathematics curriculum to life in the minds of the pupils. | a system, where resources are re-directed to older pupils at the expense of other groups and more proactive approaches. Behaviour and motivation of pupils who have spent a long time falling behind may be particularly impervious to correctional curricular approaches deployed during the last academic year. |

## Glossary

| Term | Description |
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| **Automaticity** | Ability to recall and deploy (facts, concepts, and methods) with **accuracy and speed** and without using conscious memory. Frees working memory for higher-order processes that require holding a line of thought. |
| **Components** | The building blocks or sub-skills a pupil needs to know to know and recall from long-term memory in order to be successful in a complex task. See Automaticity. |
| **Composites** | The more complex knowledge which can be acquired or more complex tasks which can be undertaken when prior knowledge components are secure in a pupil's memory. |
| **Conceptual knowledge/'type II' knowledge** | Awareness, adaptability and ability to choose relevant mathematics; knowledge of links/relationships within and between knowledge. |
| **Conditional knowledge** | Combinations of procedural & declarative knowledge and their pairings to conditions. Used to solve novel, but familiar problems. |
| **Cumulative dysfluency** | Educational failure caused when pupils do not have enough opportunities to recall knowledge to gain automaticity with the use of that knowledge. Over time this may prevent or limit pupils' acquisition of more complex knowledge. |
| **Cumulative sufficiency** | Sufficient coverage of significant topics in the curriculum, particularly where identification of individually significant topics is not possible. |
| **Declarative knowledge** | Examples: facts, concepts, rules & formulae. Any aspect of knowledge that can be recalled and used with the sentence stem 'I know that'. |
| **Deep structure** | Of problems: what is left after extraneous information is rule out, and relevant information is converted into expressions and equations. |
| **Derivation** | The ability to combine known facts and concepts to summon new facts and concepts. |
| **Disciplinary knowledge** | Methods and conceptual frameworks used by specialists in a given subject, e.g. knowledge of history or geography as a discipline. |
| **Exercises** | Sets of questions where the solution is not known, but the procedure is pre-defined. Usually used for discrete practice to make procedural knowledge firm and precise in the mind before moving onto related problem solving. |

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| **Fluency** | Varied meanings:<br>▪ the speed and accuracy of recall of facts and methods. This document will use the term 'automaticity' as an approximation of this idea<br>▪ the ability to make links between concepts often referred to as 'mathematical understanding'. This document will use the term 'mathematical understanding', to refer to the idea of adaptable use of mathematics<br>This document will not use the word fluency because it is used to mean different things within mathematics education and confusion can be avoided using other terms. |
| **Inflexible knowledge** | (New) knowledge that is yet to form links with related knowledge in the mathematician's mind; pupils are not as familiar with all the ways it can be used and all the ways it is connected to other knowledge. |
| **Mastery** | Varied meanings:<br>▪ deep understanding of concept/topic<br>▪ a scheme of learning<br>▪ proficiency in a topic area acquired through purposeful practice<br>This document will not use the word mastery because it is used to mean different things within mathematics education and confusion can be avoided using other terms. |
| **Mathematical understanding** | Varied meanings:<br>▪ ability to make sense of something because of its links with what is already known<br>▪ (not used here) proof of learning |
| **Memory** | ▪ working, or short-term memory: where conscious processing or 'thoughts' occur. Limited to holding 4 to 7 items of information for up to around 30 seconds at a time.<br>▪ long-term memory: where knowledge resides in integrated schema, ready for use without taking up working memory |
| **Phonics** | The study of the relationship between the spoken and written language. Each letter or combination of letters represent a sound or sounds. The information is codified, as we must be able to recognise which symbols make which sounds in order to read the language. |

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| **Problem solving** | Varied meanings:<br>▪ The act of solving a familiar problem(s) for which a strategy has already been taught, learned and applied<br>▪ An experience intended to give pupils a sense of what real mathematicians do*<br>▪ A generic experience where collections of questions are diverse both in terms of surface features and deep structures, designed to promote learning of generic problem-solving 'skills'**<br>*unlikely to be possible as pupils do not have the expert knowledge to conjecture and reason at the frontier of mathematics<br>**research informs us that generic problem-solving skills cannot be taught/learned, as expertise is domain/topic specific |
| **Problems** | Broadly split into two categories:<br>▪ Within a subclass, underpinned by conditional knowledge: familiar questions where the solution is not known and a strategy needs to be matched with the deep structure of the problem<br>▪ Open-ended: unfamiliar questions where the solution is not known and pupils must work out novel strategies<br>Sometimes refers to (in schools):<br>▪ Questions requiring the use of algebraic manipulation<br>▪ Worded/'real life' problems |
| **Procedural knowledge** | Core algorithms, methods and procedures. |
| **Progression model** | The planned path from the pupil's current state of mathematical competence to the school's intended manifestation of expertise. |
| **Purposeful practice** | An activity that requires the deployment of a new, known procedure, but where questions vary slightly throughout the collection of questions to shine a light on new patterns, relationships, connections, rules and concepts that then become firm and precise in the mind. |

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| **Reasoning** | Manifestation of type II conditional knowledge of relationships between strategy, known and unknown information – knowing *why* a strategy would work in a situation. |
| **Schema/schemata (plural)** | A pattern of thought that organises categories of information and the connections between them. |
| **Substantive knowledge** | Significant concepts; knowledge that carries considerable weight in a given subject domain, eg concepts such as 'empire', 'civilisation', 'parliament' and 'peasantry' in history. |
| **Symbolic abstraction** | The process of converting the deep structure of a question into expressions and equations that can then be used to find the value of unknown variable(s). |